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An Experimental Analysis of Adaptive Learning in a Multi-Subject Economy

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Abstract

The rational expectations hypothesis (REH) has long served as a foundation in macroeconomic laws of motion. However, the assumptions of REH are likely too powerful to be representative of economic actors. This research evaluates adaptive learning, a developing alternative to rational expectations, using a multi-agent macroeconomic prediction "game." Data was gathered from a group of students, each predicting the outcome of a single economy over time. Each agent was asked to forecast output (GDP) and inflation in each period based on historic levels of output, inflation, and interest rates. These data were then analyzed under various theoretical models of adaptive learning for mathematical fit.

Contents

1 Introduction

Economics has long recognized and attempted to quantify the phenomena of forward thought. Known as "expectations" in literature, has become an essential component of macroeconomic laws of motion, the dynamic mathematical systems by which macroeconomies are modeled. Researchers recognize that awareness of the future impacts how individuals make economic decisions, but have yet to determine a way to accurately portray these decisions in models. Historically, several methods have been used to approximate expectations' impact on the macroeconomy. Originally, naïve expectations were used as a simple stand-in for awareness of expectations, and the Rational Expectation Hypothesis was later developed to capture more nuanced subjects. The history and descriptions of both of these methods will be presented, as well a brief introduction to adaptive learning, the expectation formation method at the center of this paper. Additionally, this introduction will discuss how central banks may attempt to improve knowledge within a learning model through central bank communication.

"Naïve" expectations were the original representation of an agents consideration of future economic conditions. It can be described as follows:

$$x^e_{(t+1)} = x_t$$

where x_t is economic output in the t^{th} time period. The superscripted term, $x^e_{(t+1)}$ represents the expected value of growth in the following time period. While this assumption captures an awareness of the future, it is too weak to accurately model the complexity of human inductive reasoning, as economic agents are able to recognize that tomorrow's growth may bring different outcomes than did today's. Then, following the work of Muth (1961) and Lucas (1967), the Rational Expectations Hypothesis (REH), became the standard for modeling macroeconomic expectations, and has remained the driving representation in macroeconomic research and modeling to date.

In a deterministic model, REH can be simply stated:

$$x^e_{(t+1)} + g_{(t+1)} = x_{(t+1)}$$

Powerful and mathematically elegant, REH led to significant developments in economic theory and understanding. Under this framework, a "rational" agent has perfect understanding of the underlying laws of motion of an economic system, allowing each agent to have a near complete view of the systems evolution. Only random economic shocks, represented by g above, are outside of the pool of knowledge available to a rational agent. Random shocks are events outside of market interaction which can have an impact on growth or inflation, such as fluctuations in oil prices or natural disasters. However, REH is built on several assumptions, some of which are too strong to be accurate to reality. Chief among these is the assumption that rational agents have knowledge of all exogenous variables present in the economy's laws of motion, values which scientists and industry professional must research thoroughly to even hypothesize. Furthermore, REH assumes that all agents are likeminded in their analysis of the macroeconomy and make identical forecasts of the future.

More recently, researchers have been pursuing a new method of expectation representation, known as adaptive learning, or simply "learning." Under this set of assumptions, an economic agent is believed to gain knowledge of the laws of motion over time, becoming more able to accurately forecast as they gather more experience. No singular means of representing learning has been developed, and so learning theorists have worked through experimentation to find models and trends that match how human subjects predict, perform, and adjust in economic simulation . This paper follows the work of other researchers who have created simulations of pricing models, but instead applies these simulation theories and methods to a macroeconomic framework. It builds on the Honors research of Mr. Atticus Graven '14, but delves into more realistic economic representation by gathering data from a multiple-agent economy, rather than many single-agent systems.

This paper seeks to explore various possible models of adaptive learning. By gathering data using a forecasting "game" in a simulated economy, the paper establishes a set of data to capture how subjects use their knowledge of prior outcomes to predict economic changes. Once this data set is collected, seven theoretical models of learning are tested against it to find patterns of fit and relative strength. While much discussion of various rules exists in the literature, relatively little directly tests experimental data against instictive learning processes. In not setting or prepping subjects in how directly to think about problems, the research allows for testing a sample of individuals to see if any single theoretical model proves most effective in representing expectations. Through the course of the experiment, there is a degree of inconsistency among the data, but some definitive trends appear in how the subjects may form expectations as a group.

This paper will first present relevant literature to adaptive learning. Next, it will discuss and explain the laws of motion used in the generation of the simulated economy, as well as provided an explanation of the experimental design. Afterwords will follow an explanation of the theoretical learning rules being tested against the experiental data, and the statistical outcomes of those rule tests. A discussion of the results and their implications will follow, with suggestions of improvements for future experiments with regards to consistency in the data set.

2 Literature Review

There has been a growing consensus among economists that REH contains assumptions that are too powerful to accurately represent society, as actual economic agents cannot maintain the level of information necessary to form perfectly rational predictions. The complexity of this problem is addressed in Arthur (1992), who suggests the problems reach a level of complexity after which computational and deductive solutions are impossible to find. Arthur argues that these questions beyond the "Problem Complexity Boundary" must therefore be solve inductively, drawing on prior knowledge and applying it to a current issue. This form of problem solving has been used mathematically for many years, and can be represented formulaically. Arthur further suggests that this form of reasoning could be tested and specified using experimental methods, which have since been employed in forecasting research.

Forecasting models in expectation formation follow a significant form throughout research. They are generally set up as a forecasting "game," in which subjects are given some set of information about the economic system. Based on the provided information, each subject is asked to forecast the value of some endogenous variable in the next time period. These forecasted values are then used to generate an "expectation value," which is input into a system of equations to calculate the actualized values of the variable given the input expectations. Subjects are presented with the new period's data, and are asked to forecast again with the new information. This process continues for a given number of periods, and often subjects are compensated based on their prediction accuracy.

Many examinations of expectation formation have developed through asset pricing systems, wherein subjects attempt to predict the value of some tradable good. Marimon and Sunder (1993) created a test environment in which subjects spent some periods engaged in trading goods, and other times outside the transactions focused on price prediction. Again, prices were shown to deviate from rational equilibrium, with more consistent with adaptive learning as a behavioral basis. Hommes, Sonnemans, Tuinstra and van de Velden (2005) took a more classic example of such a model, using subjects with information of dividend values and interest rates to predict the value of a good against several pre-programmed, "fundamentalist" traders which always predicted a rational price. This test found that, in general, simple adaptive learning rules were more effective than the REH in determining price levels of the asset. This paper, while it targets similar goals of analyzing expectation formation, approaches the issue through a macroeconomic framework. This is done to eliminate the aspects of dividend payments and other extraneous factors, and instead focus on the expectation values assumed in the Dynamic Schocastic Equilibrium (DSGE) models which have become a central instrument of macroeconomic theory. Many of these models assume REH subjects as a foundational aspect of their equilibria (Woodford 2003), but the validity of such restricting ideas fails to support recent empirical evidence. By tackling these models directly, this paper hopes to find some evidence to match macroeconomic theory with more recent empirical findings.

The macroeconomic forecasting model this paper uses is based in part on the work of Graven (2014), but varies from his work and other literature most notably in the difference in size of experimental economies. Graven (2014) relied on single-subject economy data, allowing each participant total control of the expectation terms of their system. While beneficial for his focus on parameter estimation within the system, this paper hopes to generate data more accurate to an existing economy, and thus ran tests in a multi-subject system. Furthermore, the test groups of the experiments presented here (numbering thirteen subjects and nine, respectively) are larger groups than both Hommes et al. (2005), which used forecast groups of six, and Marimon and Sunder (1993), with four forecasters. By enlarging the number of participants, the relevant weight of each expectation on the economy was lessoned, allowing for a more realistic model.

3 Experimental Design and Model

3.1 NK Model and Parameters

In order to test the various possible adaptive learning models, a forecasting "game" was developed. This "game" simulated an economy using a New-Kenysian (NK) system of equations to represent the economic laws of motion. Full details of the particular model and foundations can be found in Woodford (2003). The monetary policy rule was used for capturing expectations previously in Assenza, Heemeijer, Hommes and Massaro (2013). This Dynamic Schocastic Equilibrium (DSGE) Model is built on micro-foundations of representative agents which are fully utility-maximizing and firm-maximizing representative firms, and was previously used in Graven (2014). The model's laws of motion can be displayed as follows:

$$x_t = x_{t+1}^e - \phi(i_t - \pi_{t+1}^e) + g_t$$
$$\pi_t = \beta\pi_{t+1}^e + \lambda x_t + u_t$$
$$i_t = \bar{\pi} + \theta_\pi(\pi_t - \bar{\pi}) + \varepsilon_t$$

where g_t, u_t, ε_t are autocorrelated error terms of the form:

$$g_t = \delta g_{t-1} + \tilde{g}_t$$
$$u_t = \mu u_{t-1} + \tilde{u}_t$$

and $\tilde{g}_t$ and $\tilde{u}_t$ are stochastic error terms of mean 0 and standard deviation of 0.2.

These laws of motion are used to systematically replicate the standard business cycle. They are based around three endogenous variables: x_t, π_t, and i_t.

- x_t is the **output gap**. This is the standardized measure between the actual Gross Domestic Product (GDP, or "output") and its calculated natural value.

The natural value is generated assuming total employment and full production in a given year. A positive value indicates that the economy has generated more output than the natural value, while a negative value indicates worse than natural productivity. The equation above states that the output gap in period t is based on expectations of the output gap in period $t + 1$, the expected inflation rate, and the interest rate. Expectations of both the output gap and the interest rate are positively correlated with actual output. The interest rate is negatively correlated.

- π_t is the **inflation rate**, which measures the percentage change in price levels across the economy. Each period's inflation is based on inflation expectations and the period's output gap. There is a positive relationship between the expectations of both output and inflation expectations with the actual inflation rate.

- i_t is the **interest rate**, which measures the cost of borrowing money. It is influenced by a central bank, such as the United States' Federal Reserve through the use of the federal funds rate.

In each of these instances, the related random shocks can either positively or negatively impact the variables. This model assumes a two-period information lag in expectation formation. That is, subjects form expectations for period $t + 1$ based on information from period $t - 1$.

The model also includes the following parameters:

- ϕ: The inter-temporal elasticity of substitution. This is a representation of how spending changes based on the expected interest rate. A higher ϕ indicates that subjects are more sensitive to rising prices. For the purpose of this model, $\phi = 6.369$.

- β: The global discount factor. The value is always between 0 and 1, and in this case the value was set as $\beta = 0.99$.

- θ_π is the interest's rates responsiveness to inflation. In this instance $\theta_\pi = 1$
- λ is the slope of the Phillips curve, which indicates the relationship between inflation and the unemployment rate. It is used here to represent the impact that increases in productivity through employment impact the inflation rate. This paper assumes $\lambda = 0.3$

3.2 Experimental Design

Two separate iterations of the experiment were run, one with 13 subjects and another with 9. In both instances, the subjects logged into a web browser as unique users on their college-issued laptops. Experimenters discussed the goals of the experiment, the normal bounds of the output gap and inflation rate, and were informed of the compensation process. It was particularly noted that compensation would vary depending on accuracy of responses, and that there would be a random selection between inflation and output error for each user to ensure they were incentivised to accurately forecast both variables. The autocorrelated shocks were randomly generated in MATLAB 2012; subjects were not informed of the degree of the shocks. Each was presented with an initial actual output gap and inflation value. From these values and based on elementary information on the relative influences of output and inflation on one another, each subject was asked to forecast the next period's output and inflation values. The subjects were not provided any view or progress of the interest rate, though they were informed that interest rates were involved in the calculation of the model. The values of the subjects were then averaged into a single expected inflation and expected output gap value. This is one of the primary differences between this experiment and Graven (2014). The use of aggregated values to create an economic system allows for a more representative economy than a single-subject system. The average value was used as the expectation terms in the model, and from these values actual output and inflation were calculated by the program. These values were then added to a graphic and chart, and were in turn used to calculate the follow period's actual values. You can see an example frame from the program showing information in Figure 1. This iterative process continued for 45 periods in Experiment 1, and 60 periods in Experiment 2.

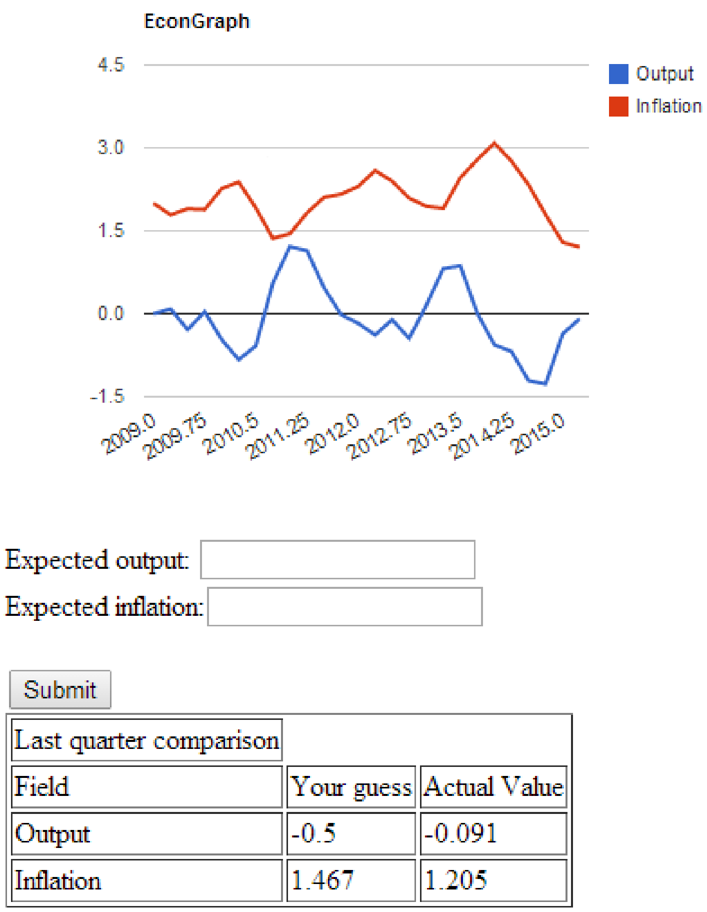

Figure 1: Sample game screen. Reproduced from Graven (2014).

4 Results

4.1 Description of Learning Rules

There are several potential learning models that have been explored in prior research. The descriptions set down in Table 1 are a selection of potential rules, adapted from Pfajfar and Zakelji (2013). Output and Inflation equations are formatted identically, with all occurences of x replaced with π and vice versa. The only exception to this rule is (6) and (7), for which the independent variables stay matched to the title of the equation. Each model is a distinct pattern of learning expression, and exist theoretically as follows:

- The autoregressive process (1) is one of the simplest hypotheses of adaptive learning, which suggests that expectations of future periods are based on the expectations of prior periods. This is an AR-1 model, meaning that only the most recent period of expectations are used in the generating of the equation.

- The sticky information method (2) assumes that otherwise rational-acting agents retain some knowledge of their previous expectation formations. This information is a convex combination of prior expectations and the REH, with λ_1 representing the weight given to the prior period's rational solution, and $1 - \lambda_1$ providing the weight of the prior expectations.

- The "true" adaptive learning model of this set, (3) assumes a constant gain rate of learning, represented by γ. This variable demonstrates the degree to which the subject uses the variance of her last expectation from the actual value to inform her expectation formation. Additionally, this rule suppresses a constant value, suggesting that all of the weight of the decision falls within the weight of the prior actual and expected values.

- A standard trend extrapolation model such as (4) suggests that subjects assume that the endogenous variable follows a continuing pattern, which she attempts to infer from the previous two periods of actual data.

- Equations (5), (6), and (7) exist as a series of models that allow for a comparative analysis. (5) is designed as a model to capture the use of all available lagged exogenous variables in decision making. (6) and (7) provide an alternative benchmark in which only output or inflation is considered.

Learning Rule	Model
(1): Autoregressive Process (π)	$\pi^e_{t+1} = \beta_0 + \beta_1 \pi^e_{t\|t-1}$
(1): Autoregressive Process (Output)	$x^e_{t+1} = \beta_0 + \beta_1 x^e_{t\|t-1}$
(2): Sticky Information (π)	$\pi^e_{t+1} = \lambda_1\beta_0 + \lambda_1\beta_1 x_{t-1} + \beta_2\pi_{t-1} + (1-\lambda_1)\pi^e_{t\|t-1}$
(2): Sticky Information (Output)	$x^e_{t+1} = \lambda_1\beta_0 + \lambda_1\beta_1 \pi_{t-1} + \beta_2 x_{t-1} + (1-\lambda_1)x^e_{t\|t-1}$
(3): Adaptive CG Learning (π)	$\pi^e_{t+1} = \pi^e_{t-1\|t-2} + \gamma(\pi_{t-1} - \pi^e_{t-1\|t-2})$
(3): Adaptive CG Learning (Output)	$x^e_{t+1} = x^e_{t-1\|t-2} + \gamma(x_{t-1} - x^e_{t-1\|t-2})$
(4): Trend Extrapolation (π)	$\pi^e_{t+1} = \beta_0 + \pi_{t-1} + \beta_1(\pi_{t-1} - \pi_{t-2})$
(4): Trend Extrapolation (Output)	$x^e_{t+1} = \beta_0 + x_{t-1} + \beta_1(x_{t-1} - x_{t-2})$
(5): General Model (π)	$\pi^e_{t+1} = \beta_0 + \beta_1\pi_{t-1} + \beta_2 x_{t-1}$
(5): General Model (Output)	$x^e_{t+1} = \beta_0 + \beta_1 x_{t-1} + \beta_2\pi_{t-1}$
(6): Lagged Output (π)	$\pi^e_{t+1} = \beta_0 + \beta_1 x_{t-1}$
(6): Lagged Output (Output)	$x^e_{t+1} = \beta_0 + \beta_1 x_{t-1}$
(7): Lagged Inflation (π)	$\pi^e_{t+1} = \beta_0 + \beta_1\pi_{t-1}$
(7): Lagged Inflation (Output)	$x^e_{t+1} = \beta_0 + \beta_1\pi_{t-1}$

Table 1: Potential learning rules, specified for inflation

Two of the models presented here deviate from those presented in Pfajfar and Zakelji (2013). (2) has been adapted to include knowledge of the lagged inflation term. A simple learning rule was judged to be too similar to (1) for the sake of this experiment, so a rational representation other than the MSV was generated to provide additional insight. In (5), the interest rate value was removed from the general model, since unlike in Pfaifar's experiment, subjects of this experiment were not exposed to the interest rate, and thus not able to factor its value into their decision-making. Each of these models provides a potential explanation for how subjects may form expectations. In addition, by comparing the relative strength of models to one another, potential trends or theoretical tests can be checked to form a fuller idea of how historic data is processed.

4.2 Statistical Analysis

The set of models discussed in the previous section were all analyzed using MLE regression analysis. Relative effectiveness of each model was evaluated through calculation of AIC terms for each model, with the lowest AIC value representing the most effective fit to the data. In order to compare models, the first three user input periods were dropped from every model to compensate the additional lagged terms used in (3) and (4). Summarized information is contained in Table 2 for inflation and Table 3 for output. Tables for each regression can be found in Appendix 2.

In evaluation of the various models, it is clear that several outcomes do not reflect the theoretical framework in which the regression analyses were created. In (3A) an impossible gain parameter is observed, as γ is larger than 1. As a gain parameter is designed to reflect the weight an agent gives to their previous estimation error, it is expected that this value should appear as an percentage term. While the model passes the linear restriction existing to ensure that the coefficient values sum to 1, a negative coefficient falls outside the reasonable values of the theoretical design. In (4) a similar issue is observed with the generation of the coefficients. In order for the regression analysis to accurately represent the theory of a constant gain learning model, the coefficients of the first and second lagged terms should be expected to have coefficents of $1 + \beta_1$ and $-\beta_1$, respectively. This linear restriction held for neither inflation nor output in either test group. Both of these models, therefore, are not found to be accurate representations of how expectations are formed among these groups. The most clearly representative model of this set, (2), also displays some degree of inconsistency within the testing. It held the best AIC numbers in three of the four test sets, and was the second most accurate model in the remaining value. However, the inflation test of experiment 1 contains an insignificant expectations term, which would suggest that rational expectations were more consistent with the regression, which doesn't match the results of the other tests in general, and not (2) in particular.

Other models followed theoretical understanding to a degree sufficient for further analysis. (1) suggested a positive relationship between lagged expectations and current expectations in each test, which refutes the suggestion of (2A)'s inflation result that rational expectations were insignificant. However, in general the autoregressive model was consistently one of the least effective models, suggesting that subjects were aware

Exp. 1/Rule	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Theory	χ				χ	χ	χ
AIC	2115.164	2022.674	2025.866	2023.558	2020.78	2133.389	2022.827
AIC Order	F	B	E	D	A	G	C
Exp. 2/Rule	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Theory	χ	χ				χ	χ
AIC	902.2884	670.6455	809.126	809.9863	824.325	1520.314	823.2568
AIC Order	F	A	B	C	E	G	D

Note: A χ indicates that the analysis is comparable to current economic theory, and is thus usable. The AIC Order provides clarification on the relative accuracy of the model, with A being the most accurate and G being the least.

Table 2: Inflation test summary across both experiments

Exp. 1/Rule	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Theory	χ	χ			χ	χ	χ
AIC	3525.75	3097.738	3142.431	3143.534	3140.239	3143.206	4260.126
AIC Order	F	A	C	D	B	E	G
Exp. 2/Rule	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Theory	χ	χ			χ	χ	χ
AIC	3531.238	3130.557	3152.654	3142.858	3141.394	3148.032	4055.597
AIC Order	F	A	E	C	B	D	G

Note: A χ indicates that the analysis is comparable to current economic theory, and is thus usable. The AIC Order provides clarification on the relative accuracy of the model, with A being the most accurate and G being the least.

Table 3: Output test summary across both experiments

of more than their former expectations when creating their forecasts.

Besides (2A) Inflation, (2) was consistently the most effective learning rule. Building on the information suggested from (1) and compared to the relative success in relation to (5), the data presented seems to indicate that there subjects used both expectations and lagged endogenous variables in their decision making, and thus used all information available to them in forecasting process. By the sticky information model we can also observed the suggested weight between prior expectations and previous actualized data. Model (2B) of inflation suggests that nearly half of the weight of the decision was based on prior expectations, but the lack of consistency with (2A) calls into question the overall validity of that observation. However, both output analyses suggested a greater weighting towards prior actualized values, with $\lambda = 0.7$ and $\lambda = 0.85$, respectively, where λ is the weighted average of these values. It is also interesting to note that both output tables suggest a negative correlation between prior inflation and expected output, which connects with the theoretical understanding of the Phillips Curve.

In our final tests we observe more specific relationships between the actualized terms of the model. It is clear from AIC values that the general model is one of the best rules empirically for every model except (5B) Inflation. It is worth noting that of these four analyses, (5B) Inflation was also the only test in which both output and inflation were not significant, with lagged output values not being an indicator of the expected inflation changes. On another note, the coefficients of the regressions respond to the presence of fuller information in the way macroeconomic theory suggests they should. In the case of (2A) inflation, the value of the prior inflation coefficient is greater in (5) than in (7), suggesting that subjects place greater value on the prior inflation value when they are synthesizing information to make predictions, rather than relying on a more naïve approach. Furthermore, in both of (5)'s output analyses, prior output is seen to have lower values when considered with prior inflation, as compared to being observed alone as in (6). This supports the assumption that subjects understand the contemporaneous relationship between higher inflation and lower output, and carry that expectation through their forecasts.

4.3 Discussion

While the data presents several insights into the potential learning rules of the subjects, there is a fair degree of inconsistency within the analyses. While the most effective models (2) and (5) tend to follow observable trends, each possesses a test which does not follow economic theory. Furthermore, these anomalies do not occur in the same experiment groups, which decreases the likelihood that the differences are due to an experimental anomaly. There are several factors which may have led to these inconsistent data. The constraints of the program limited the number of subjects of the experiment; larger data sets than those presented here were attempted, but ended prematurely due to user errors which led to server failures for the game. A larger sample size may have provided more defined separations between closely linked rules, as well as greater clarity to what occur in the inconsistent analyses. Model design may also have led to unusual outcomes. While not of direct impact to this analysis, unusual spikes in both output and inflation occured during the testing period, which may have limited the ability of the subjects to settle fully into a consistent, natural pattern of expectation formation. It is possible that these spikes were a result of the overweighting of shocks within the model, but further analysis of the laws of motion needs to be explored to ensure the stability of the DSGE setting under these conditions.

Furthermore, it is possible that there is not a single learning rule that all subjects follow. Differences in learning methods have some historic support in research, and would support a claim that learning follows is ingrained more as a desire that an iterative process. Heterogeneous decision-making on the subject level could lead to inconsistencies in aggregation. The idea of heterogeneous learning is discussed in Pfajfar and Zakelji (2013) and Hommes et al. (2005), and would be more effectively tested in a single-agent economy, or on a subject basis. Further iterations of this study could explore this subject further, but such an analysis falls outside the bounds of this paper.

5 Conclusion

This experiment was generated with the goal of examining potential learning models in an experimental setting to test their practical validity. While several trends appeared within the design, a definitive representation of expectation formation remains enigmatic. Evidence suggests that subjects favor actualized values over their prior expectations, and some analyses suggest that subjects are considering the relationship between endogenous variables, while others are not. The tests pressent evidence that the subjects of these test leaned towards a trend extrapolation rule, but the model presented in this paper is insufficient to represent their method.

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6 Appendix 1: Compensation Schedule

This compensation method adapted from Hommes, Sonnemans, Tuinstra and van de Velden (2005) and Graven (2014)

Each subject received \$5 for participating in the program, and were told that, on average, each subject could expect to receive \$15 based on their performance. It was also made clear that the compensation would be based on the error of one of their two forecasted values, selected at random. This design was to incentivise the subjects to attempt to accurately predict both variables to the best of their ability in each period. The formula is point-based, with each subject receiving a number of points each period based on the squared error of their expectation from the actual value of the period. Previous work suggests that inflation rates are easier to predict than output(see Graven (2014)), so an exchange rate was established to create a \$15 average return independent of which endogenous variable is selected. The exchange rates for inflation and output, respectively, are:

$$\rho_\pi = 4,900 \texttt{ points / dollar}$$

$$\rho_x = 600 \texttt{ points / dollar}$$

Using this point system, earnings were calculated based on a maximization equation. Let v_i be the the output gap x or the inflation rate π for subject i. Then, v_{it} is that variable in period t, and v^e is the ith subject's prediction for that variable in period t.

$$e_{it} = max(2000 - \frac{2000}{0.156}[(v - v_{it}^e)]^2, 0)$$

Then, e_{it} is the point earnings in period t of subject i. The total earnings of i can be written:

$$\frac{\Sigma_{t=0}^{T}(e_{it})}{\rho_i}$$

Where T is the total number of periods, and ρ_i is the variable's rate of exchange. In the case of this experiment, $T = 45$ for Experiment 1, and $T = 60$ for Experiment 2.

7 Appendix 2: Regression Models

Experiment 1 — Inflation Learning Rule (1) — Experiment 2

Experiment 1

```
Random-effects ML regression                    Number of obs     =        546
Group variable: id                              Number of groups  =         13

Random effects u_i ~ Gaussian                   Obs per group: min =        42
                                                               avg =      42.0
                                                               max =        42

                                                LR chi2(1)        =      21.12
Log likelihood  = -1053.5819                    Prob > chi2       =     0.0000
```

Einf	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
lagEinf	.1947835	.0419769	4.64	0.000	.1125102	.2770567
_cons	1.727177	.1146807	15.06	0.000	1.502407	1.951947
/sigma_u	0	.1151033			.	.
/sigma_e	1.666455	.0504282			1.570492	1.768282
rho	0	(omitted)				

Likelihood-ratio test of sigma_u=0: chibar2(01)= 0.00 Prob>=chibar2 = 1.000

. estat ic

Akaike's information criterion and Bayesian information criterion

Model	Obs	ll(null)	ll(model)	df	AIC	BIC
.	546	-1064.141	-1053.582	4	2115.164	2132.374

Experiment 2

```
Random-effects ML regression                    Number of obs     =        522
Group variable: id                              Number of groups  =          9

Random effects u_i ~ Gaussian                   Obs per group: min =        58
                                                               avg =      58.0
                                                               max =        58

                                                LR chi2(1)        =     863.25
Log likelihood  = -447.14418                    Prob > chi2       =     0.0000
```

Einf	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
lagEinf	.8950181	.0190547	46.97	0.000	.8576716	.9323646
_cons	.2471537	.053831	4.59	0.000	.1416469	.3526606
/sigma_u	0	.0258486			.	.
/sigma_e	.5698737	.0176367			.5363339	.605511
rho	0	(omitted)				

Likelihood-ratio test of sigma_u=0: chibar2(01)= 0.00 Prob>=chibar2 = 1.000

. estat ic

Akaike's information criterion and Bayesian information criterion

Model	Obs	ll(null)	ll(model)	df	AIC	BIC
.	522	-878.771	-447.1442	4	902.2884	919.319

Experiment 1 — Inflation Learning Rule (2) — Experiment 2

Experiment 1

```
Random-effects ML regression                    Number of obs     =        546
Group variable: id                              Number of groups  =         13

Random effects u_i ~ Gaussian                   Obs per group: min =        42
                                                               avg =      42.0
                                                               max =        42

                                                LR chi2(3)        =     117.61
Log likelihood  = -1005.3368                    Prob > chi2       =     0.0000
```

Einf	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
laggdp	.0114044	.0057144	2.00	0.046	.0002045	.0226044
laginf	.8986082	.087816	10.23	0.000	.7264919	1.070724
lagEinf	-.0146539	.0448981	-0.33	0.744	-.1026525	.0733447
_cons	.2033353	.201815	1.01	0.314	-.1922148	.5988855
/sigma_u	.0958534	.1362905			.0059063	1.555611
/sigma_e	1.52273	.0466669			1.433958	1.616999
rho	.0039469	.0112151			2.89e-06	.2180436

Likelihood-ratio test of sigma_u=0: chibar2(01)= 0.15 Prob>=chibar2 = 0.350

. estat ic

Akaike's information criterion and Bayesian information criterion

Model	Obs	ll(null)	ll(model)	df	AIC	BIC
.	546	-1064.141	-1005.337	6	2022.674	2048.489

Experiment 2

```
Random-effects ML regression                    Number of obs     =        522
Group variable: id                              Number of groups  =          9

Random effects u_i ~ Gaussian                   Obs per group: min =        58
                                                               avg =      58.0
                                                               max =        58

                                                LR chi2(3)        =    1098.90
Log likelihood  = -329.32273                    Prob > chi2       =     0.0000
```

Einf	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
laggdp	.0108321	.0017968	6.03	0.000	.0073104	.0143538
laginf	.6808355	.0415052	16.40	0.000	.5994868	.7621841
lagEinf	.493055	.0374325	13.17	0.000	.4196886	.5664213
_cons	-.4081129	.0692946	-5.89	0.000	-.5439277	-.272298
/sigma_u	.0500178	.030167			.0153371	.1631193
/sigma_e	.4526355	.0141456			.4257427	.481227
rho	.0120637	.0144607			.0007933	.0882164

Likelihood-ratio test of sigma_u=0: chibar2(01)= 1.29 Prob>=chibar2 = 0.128

. estat ic

Akaike's information criterion and Bayesian information criterion

Model	Obs	ll(null)	ll(model)	df	AIC	BIC
.	522	-878.771	-329.3227	6	670.6455	696.1915

Experiment 1 — Inflation Learning Rule (3) — Experiment 2

Experiment 1

```
Random-effects ML regression                    Number of obs      =       546
Group variable: id                              Number of groups   =        13

Random effects u_i ~ Gaussian                   Obs per group: min =        42
                                                               avg =      42.0
                                                               max =        42

                                                Wald chi2(2)       =    693.66
Log likelihood  = -1008.9328                    Prob > chi2        =    0.0000
```

Einf	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
lag2Einf	-.0745338	.041971	-1.78	0.076	-.1567954	.0077278
laginf	1.064198	.0525667	20.24	0.000	.9611696	1.167227
/sigma_u	.1626821	.1141835			.0411053	.6438461
/sigma_e	1.528506	.0469918			1.439124	1.62344
rho	.0112009	.015653			.00044	.1073638

```
Likelihood-ratio test of sigma_u=0: chibar2(01)=     0.83 Prob>=chibar2 = 0.181

.  test lag2Einf = 1-laginf

 ( 1)  [Einf]lag2Einf + [Einf]laginf = 1

           chi2(  1) =    0.07
         Prob > chi2 =    0.7882

. estat ic

Akaike's information criterion and Bayesian information criterion
```

Model	Obs	ll(null)	ll(model)	df	AIC	BIC
.	546	.	-1008.933	4	2025.866	2043.076

Experiment 2

```
Random-effects ML regression                    Number of obs      =       522
Group variable: id                              Number of groups   =         9

Random effects u_i ~ Gaussian                   Obs per group: min =        58
                                                               avg =      58.0
                                                               max =        58

                                                Wald chi2(2)       =   3070.55
Log likelihood  = -400.56302                    Prob > chi2        =    0.0000
```

Einf	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
lag2Einf	.1220501	.0280846	4.35	0.000	.0670052	.177095
laginf	.9485537	.0341157	27.80	0.000	.8816882	1.015419
/sigma_u	.166346	.0555648			.0864339	.3201402
/sigma_e	.5124835	.0160668			.4819411	.5449614
rho	.0953152	.058222			.0237271	.2626458

```
Likelihood-ratio test of sigma_u=0: chibar2(01)=    19.75 Prob>=chibar2 = 0.000

.  test lag2Einf = 1-laginf

 ( 1)  [Einf]lag2Einf + [Einf]laginf = 1

           chi2(  1) =   13.27
         Prob > chi2 =    0.0003

. estat ic

Akaike's information criterion and Bayesian information criterion
```

Model	Obs	ll(null)	ll(model)	df	AIC	BIC
.	522	.	-400.563	4	809.126	826.1567

Experiment 1 — Inflation Learning Rule (4) — Experiment 2

Experiment 1

```
Random-effects ML regression                    Number of obs      =       546
Group variable: id                              Number of groups   =        13

Random effects u_i ~ Gaussian                   Obs per group: min =        42
                                                               avg =      42.0
                                                               max =        42

                                                LR chi2(2)         =    114.72
Log likelihood  = -1006.7789                    Prob > chi2        =    0.0000
```

Einf	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
lag2inf	-.145267	.1288696	-1.13	0.260	-.3978469	.1073129
laginf	.9515073	.1288635	7.38	0.000	.6989394	1.204075
_cons	.4743093	.1747774	2.71	0.007	.131752	.8168666
/sigma_u	.083776	.1478617			.002635	2.663547
/sigma_e	1.527393	.0467813			1.438401	1.621891
rho	.0029994	.0105826			2.65e-07	.3154709

```
Likelihood-ratio test of sigma_u=0: chibar2(01)=     0.09 Prob>=chibar2 = 0.380

. test 1+laginf = -lag2inf

 ( 1)  [Einf]lag2inf + [Einf]laginf = -1

           chi2(  1) =  542.13
         Prob > chi2 =    0.0000

. estat ic

Akaike's information criterion and Bayesian information criterion
```

Model	Obs	ll(null)	ll(model)	df	AIC	BIC
.	546	-1064.141	-1006.779	5	2023.558	2045.071

Experiment 2

```
Random-effects ML regression                    Number of obs      =       522
Group variable: id                              Number of groups   =         9

Random effects u_i ~ Gaussian                   Obs per group: min =        58
                                                               avg =      58.0
                                                               max =        58

                                                LR chi2(2)         =    957.56
Log likelihood  = -399.99314                    Prob > chi2        =    0.0000
```

Einf	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
lag2inf	-.2380305	.0604582	-3.94	0.000	-.3565264	-.1195345
laginf	1.318348	.0600949	21.94	0.000	1.200565	1.436132
_cons	-.1218549	.0743368	-1.64	0.101	-.2675524	.0238426
/sigma_u	.146788	.0418912			.0839016	.2568095
/sigma_e	.5128616	.0160103			.4824227	.545221
rho	.0757158	.0402823			.0233939	.1892209

```
Likelihood-ratio test of sigma_u=0: chibar2(01)=    25.36 Prob>=chibar2 = 0.000

. test 1+laginf = -lag2inf

 ( 1)  [Einf]lag2inf + [Einf]laginf = -1

           chi2(  1) = 9669.75
         Prob > chi2 =    0.0000

. estat ic

Akaike's information criterion and Bayesian information criterion
```

Model	Obs	ll(null)	ll(model)	df	AIC	BIC
.	522	-878.771	-399.9931	5	809.9863	831.2746

Experiment 1 — Inflation Learning Rule (5) — Experiment 2

Experiment 1

Random-effects ML regression — Number of obs = 546
Group variable: **id** — Number of groups = 13

Random effects u_i ~ **Gaussian** — Obs per group: min = 42; avg = 42.0; max = 42

LR chi2(**2**) = 117.50
Log likelihood = **-1005.39** — Prob > chi2 = 0.0000

Einf	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
laginf	.885635	.0783406	11.30	0.000	.7320902	1.03918
laggdp	.0115057	.0057085	2.02	0.044	.0003172	.0226942
_cons	.197847	.2008426	0.99	0.325	-.1957973	.5914914
/sigma_u	.0854819	.1448968			.0030835	2.369795
/sigma_e	1.523418	.0466595			1.434658	1.61767
rho	.0031387	.0106349			4.32e-07	.2926923

Likelihood-ratio test of sigma_u=0: chibar2(01)= **0.10** Prob>=chibar2 = **0.375**

. **estat ic**

Akaike's information criterion and Bayesian information criterion

Model	Obs	ll(null)	ll(model)	df	AIC	BIC
.	546	-1064.141	-1005.39	5	2020.78	2042.293

Experiment 2

Random-effects ML regression — Number of obs = 522
Group variable: **id** — Number of groups = 9

Random effects u_i ~ **Gaussian** — Obs per group: min = 58; avg = 58.0; max = 58

LR chi2(**2**) = 943.22
Log likelihood = **-407.16249** — Prob > chi2 = 0.0000

Einf	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
laginf	1.115474	.0289268	38.56	0.000	1.058779	1.17217
laggdp	.0018444	.0019098	0.97	0.334	-.0018988	.0055876
_cons	-.211771	.0897692	-2.36	0.018	-.3877154	-.0358267
/sigma_u	.1463495	.0420173			.0833699	.2569055
/sigma_e	.5200793	.0162356			.489212	.5528941
rho	.0733749	.0393707			.0224586	.1849162

Likelihood-ratio test of sigma_u=0: chibar2(01)= **24.29** Prob>=chibar2 = **0.000**

. **estat ic**

Akaike's information criterion and Bayesian information criterion

Model	Obs	ll(null)	ll(model)	df	AIC	BIC
.	522	-878.771	-407.1625	5	824.325	845.6133

Experiment 1 — Inflation Learning Rule (6) — Experiment 2

Experiment 1

Random-effects ML regression — Number of obs = 546
Group variable: **id** — Number of groups = 13

Random effects u_i ~ **Gaussian** — Obs per group: min = 42; avg = 42.0; max = 42

LR chi2(**1**) = 2.89
Log likelihood = **-1062.6947** — Prob > chi2 = 0.0890

Einf	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
laggdp	-.0101838	.0059802	-1.70	0.089	-.0219048	.0015371
_cons	2.243694	.0932319	24.07	0.000	2.060963	2.426425
/sigma_u	0	.2489747			.	.
/sigma_e	1.694502	.0512769			1.596923	1.798043
rho	0	(omitted)				

Likelihood-ratio test of sigma_u=0: chibar2(01)= **0.00** Prob>=chibar2 = **1.000**

. **estat ic**

Akaike's information criterion and Bayesian information criterion

Model	Obs	ll(null)	ll(model)	df	AIC	BIC
.	546	-1064.141	-1062.695	4	2133.389	2150.6

Experiment 2

Random-effects ML regression — Number of obs = 522
Group variable: **id** — Number of groups = 9

Random effects u_i ~ **Gaussian** — Obs per group: min = 58; avg = 58.0; max = 58

LR chi2(**1**) = 245.23
Log likelihood = **-756.15692** — Prob > chi2 = 0.0000

Einf	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
laggdp	-.0486599	.0027446	-17.73	0.000	-.0540393	-.0432806
_cons	2.557342	.0539755	47.38	0.000	2.451552	2.663132
/sigma_u	.0888868	.0694534			.0192192	.4110925
/sigma_e	1.026882	.032058			.9659332	1.091677
rho	.0074369	.0115869			.0001999	.0916546

Likelihood-ratio test of sigma_u=0: chibar2(01)= **0.65** Prob>=chibar2 = **0.210**

. **estat ic**

Akaike's information criterion and Bayesian information criterion

Model	Obs	ll(null)	ll(model)	df	AIC	BIC
.	522	-878.771	-756.1569	4	1520.314	1537.345

Experiment 1 — Inflation Learning Rule (7) — Experiment 2

Experiment 1

Random-effects ML regression — Number of obs = 546
Group variable: **id** — Number of groups = 13

Random effects u_i ~ **Gaussian** — Obs per group: min = 42, avg = 42.0, max = 42

LR chi2(**1**) = 113.45
Log likelihood = **-1007.4135** — Prob > chi2 = 0.0000

Einf	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
laginf	.8325668	.0740641	11.24	0.000	.6874038	.9777298
_cons	.4204375	.1682827	2.50	0.012	.0906095	.7502655
/sigma_u	.0829819	.1492834			.0024416	2.820227
/sigma_e	1.529213	.046837			1.440115	1.623823
rho	.002936	.0105588			2.08e-07	.3268882

Likelihood-ratio test of sigma_u=0: chibar2(01)= **0.09** Prob>=chibar2 = **0.382**

. **estat ic**

Akaike's information criterion and Bayesian information criterion

Model	Obs	ll(null)	ll(model)	df	AIC	BIC
.	546	-1064.141	-1007.413	4	2022.827	2040.037

Experiment 2

Random-effects ML regression — Number of obs = 522
Group variable: **id** — Number of groups = 9

Random effects u_i ~ **Gaussian** — Obs per group: min = 58, avg = 58.0, max = 58

LR chi2(**1**) = 942.29
Log likelihood = **-407.62838** — Prob > chi2 = 0.0000

Einf	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
laginf	1.096317	.0210731	52.02	0.000	1.055014	1.137619
_cons	-.1628183	.0741209	-2.20	0.028	-.3080926	-.0175439
/sigma_u	.1463205	.0420257			.0833347	.2569123
/sigma_e	.5205518	.0162509			.4896555	.5533977
rho	.0732246	.0393121			.0223987	.1846396

Likelihood-ratio test of sigma_u=0: chibar2(01)= **24.22** Prob>=chibar2 = **0.000**

. **estat ic**

Akaike's information criterion and Bayesian information criterion

Model	Obs	ll(null)	ll(model)	df	AIC	BIC
.	522	-878.771	-407.6284	4	823.2568	840.2874

Experiment 1 — Output Learning Rule (1) — Experiment 2

Experiment 1

Random-effects ML regression — Number of obs = 546
Group variable: **id** — Number of groups = 13

Random effects u_i ~ **Gaussian** — Obs per group: min = 42, avg = 42.0, max = 42

LR chi2(1) = 803.87
Log likelihood = **-1758.8751** — Prob > chi2 = 0.0000

Egdp	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
lagEgdp	.8789008	.0205222	42.83	0.000	.8386781	.9191235
_cons	1.152497	.3269436	3.53	0.000	.5116996	1.793295
/sigma_u	0	.2624463			.	.
/sigma_e	6.064454	.1835148			5.71523	6.435016
rho	0	(omitted)				

Likelihood-ratio test of sigma_u=0: chibar2(01)= **0.00** Prob>=chibar2 = **1.000**

. **estat ic**

Akaike's information criterion and Bayesian information criterion

Model	Obs	ll(null)	ll(model)	df	AIC	BIC
.	546	-2160.809	-1758.875	4	3525.75	3542.961

Experiment 2

Random-effects ML regression — Number of obs = 522
Group variable: **id** — Number of groups = 9

Random effects u_i ~ **Gaussian** — Obs per group: min = 58, avg = 58.0, max = 58

LR chi2(1) = 849.77
Log likelihood = **-1761.6188** — Prob > chi2 = 0.0000

Egdp	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
lagEgdp	.8969533	.0194042	46.22	0.000	.8589216	.9349849
_cons	.3540368	.3128884	1.13	0.258	-.2592132	.9672868
/sigma_u	0	.3108673			.	.
/sigma_e	7.069644	.2187949			6.65356	7.511747
rho	0	(omitted)				

Likelihood-ratio test of sigma_u=0: chibar2(01)= **0.00** Prob>=chibar2 = **1.000**

. **estat ic**

Akaike's information criterion and Bayesian information criterion

Model	Obs	ll(null)	ll(model)	df	AIC	BIC
.	522	-2186.504	-1761.619	4	3531.238	3548.268

Experiment 1 — Output Learning Rule (2) — Experiment 2

```
Random-effects ML regression                    Number of obs     =        546
Group variable: id                              Number of groups  =         13

Random effects u_i ~ Gaussian                   Obs per group: min =         42
                                                               avg =       42.0
                                                               max =         42

                                                LR chi2(3)        =    1235.88
Log likelihood  =  -1542.869                    Prob > chi2       =     0.0000

------------------------------------------------------------------------------
        Egdp |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      laggdp |   .6737468   .0465544    14.47   0.000     .5825018    .7649918
      laginf |  -1.584764   .2653321    -5.97   0.000    -2.104805   -1.064722
     lagEgdp |   .2851955   .0424231     6.72   0.000     .2020478    .3683433
       _cons |   3.583801   .6596541     5.43   0.000     2.290903      4.8767
-------------+----------------------------------------------------------------
    /sigma_u |   .5574225   .2593996                      .2239099      1.3877
    /sigma_e |   4.054683   .1243274                      3.818183    4.305831
         rho |   .0185491   .0170878                      .0023854    .0889506
------------------------------------------------------------------------------
Likelihood-ratio test of sigma_u=0: chibar2(01)=     2.30 Prob>=chibar2 = 0.065

. estat ic

Akaike's information criterion and Bayesian information criterion

-----------------------------------------------------------------------------
       Model |    Obs    ll(null)   ll(model)     df          AIC         BIC
-------------+---------------------------------------------------------------
           . |    546   -2160.809   -1542.869      6     3097.738    3123.554
-----------------------------------------------------------------------------
```

```
Random-effects ML regression                    Number of obs     =        522
Group variable: id                              Number of groups  =          9

Random effects u_i ~ Gaussian                   Obs per group: min =         58
                                                               avg =       58.0
                                                               max =         58

                                                LR chi2(3)        =    1286.14
Log likelihood  = -1559.2787                    Prob > chi2       =     0.0000

------------------------------------------------------------------------------
        Egdp |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      laggdp |    .722192   .0502653    14.37   0.000     .6236739    .8207102
      laginf |  -1.374779    .311753    -4.41   0.000    -1.985804   -.7637547
     lagEgdp |   .1570066   .0437389     3.59   0.000     .0712799    .2427333
       _cons |   4.416415   .7840471     5.63   0.000     2.879711    5.953119
-------------+----------------------------------------------------------------
    /sigma_u |   .2616027    .431135                      .0103471    6.614045
    /sigma_e |   4.791342   .1496467                      4.506837    5.093808
         rho |   .0029722   .0097907                      5.70e-07    .2624635
------------------------------------------------------------------------------
Likelihood-ratio test of sigma_u=0: chibar2(01)=     0.11 Prob>=chibar2 = 0.369

. estat ic

Akaike's information criterion and Bayesian information criterion

-----------------------------------------------------------------------------
       Model |    Obs    ll(null)   ll(model)     df          AIC         BIC
-------------+---------------------------------------------------------------
           . |    522   -2202.347   -1559.279      6     3130.557    3156.103
-----------------------------------------------------------------------------
```

Experiment 1 — Output Learning Rule (3) — Experiment 2

```
Random-effects ML regression                    Number of obs     =        546
Group variable: id                              Number of groups  =         13

Random effects u_i ~ Gaussian                   Obs per group: min =         42
                                                               avg =       42.0
                                                               max =         42

                                                Wald chi2(2)      =    5278.41
Log likelihood  = -1567.2155                    Prob > chi2       =     0.0000

------------------------------------------------------------------------------
        Egdp |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    lag2Egdp |   .0202732   .0227148     0.89   0.372    -.0242471    .0647934
      laggdp |    .965241   .0233966    41.26   0.000     .9193844    1.011098
-------------+----------------------------------------------------------------
    /sigma_u |   .9489301   .2757327                      .5369066    1.677142
    /sigma_e |   4.211535   .1290145                      3.966113    4.472144
         rho |   .0483149   .0270139                      .0142972     .128439
------------------------------------------------------------------------------
Likelihood-ratio test of sigma_u=0: chibar2(01)=    11.81 Prob>=chibar2 = 0.000

.  test lag2Egdp = 1-laggdp

 ( 1)  [Egdp]lag2Egdp + [Egdp]laggdp = 1

           chi2(  1) =    1.07
         Prob > chi2 =    0.3007

. estat ic

Akaike's information criterion and Bayesian information criterion

-----------------------------------------------------------------------------
       Model |    Obs    ll(null)   ll(model)     df          AIC         BIC
-------------+---------------------------------------------------------------
           . |    546           .   -1567.215      4     3142.431    3159.641
-----------------------------------------------------------------------------
```

```
Random-effects ML regression                    Number of obs     =        522
Group variable: id                              Number of groups  =          9

Random effects u_i ~ Gaussian                   Obs per group: min =         58
                                                               avg =       58.0
                                                               max =         58

                                                Wald chi2(2)      =    5139.93
Log likelihood  =  -1572.327                    Prob > chi2       =     0.0000

------------------------------------------------------------------------------
        Egdp |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    lag2Egdp |  -.0610176   .0256887    -2.38   0.018    -.1113665   -.0106688
      laggdp |   .9790147   .0249355    39.26   0.000     .9301419    1.027887
-------------+----------------------------------------------------------------
    /sigma_u |   1.373035   .3957694                        .780419    2.415659
    /sigma_e |   4.846448   .1513194                        4.55876    5.152291
         rho |   .0742997   .0399887                        .022634    .1875832
------------------------------------------------------------------------------
Likelihood-ratio test of sigma_u=0: chibar2(01)=    23.60 Prob>=chibar2 = 0.000

.  test lag2Egdp = 1-laggdp

 ( 1)  [Egdp]lag2Egdp + [Egdp]laggdp = 1

           chi2(  1) =   35.94
         Prob > chi2 =    0.0000

. estat ic

Akaike's information criterion and Bayesian information criterion

-----------------------------------------------------------------------------
       Model |    Obs    ll(null)   ll(model)     df          AIC         BIC
-------------+---------------------------------------------------------------
           . |    522           .   -1572.327      4     3152.654    3169.685
-----------------------------------------------------------------------------
```

Experiment 1 — Output Learning Rule (4) — Experiment 2

```
Random-effects ML regression                    Number of obs      =       546
Group variable: id                              Number of groups   =        13

Random effects u_i ~ Gaussian                   Obs per group: min =        42
                                                               avg =      42.0
                                                               max =        42

                                                LR chi2(2)         =   1188.08
Log likelihood  = -1566.767                     Prob > chi2        =    0.0000

------------------------------------------------------------------------------
        Egdp |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     lag2gdp |   .3135448    .2422811     1.29   0.196    -.1613174    .7884069
      laggdp |    .993743    .0176537    56.29   0.000     .9591424    1.028344
       _cons |  -.7166925    .6929315    -1.03   0.301    -2.074813    .6414282
-------------+----------------------------------------------------------------
    /sigma_u |      .9782    .2765945                      .5620088    1.702598
    /sigma_e |   4.205893    .1288117                      3.960854     4.46609
         rho |   .0513169    .0278333                      .0157651    .1326141
------------------------------------------------------------------------------
Likelihood-ratio test of sigma_u=0: chibar2(01)=    13.35 Prob>=chibar2 = 0.000

. test 1+laggdp = -lag2gdp

 ( 1)  [Egdp]lag2gdp + [Egdp]laggdp = -1

           chi2(  1) =   83.65
         Prob > chi2 =    0.0000

. estat ic

Akaike's information criterion and Bayesian information criterion

-----------------------------------------------------------------------------
       Model |    Obs    ll(null)   ll(model)     df          AIC         BIC
-------------+---------------------------------------------------------------
           . |    546   -2160.809   -1566.767      5     3143.534    3165.047
-----------------------------------------------------------------------------
```

```
Random-effects ML regression                    Number of obs      =       522
Group variable: id                              Number of groups   =         9

Random effects u_i ~ Gaussian                   Obs per group: min =        58
                                                               avg =      58.0
                                                               max =        58

                                                LR chi2(2)         =   1240.15
Log likelihood  = -1566.4289                    Prob > chi2        =    0.0000

------------------------------------------------------------------------------
        Egdp |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     lag2gdp |  -.9070742    .3374575    -2.69   0.007    -1.568479   -.2456696
      laggdp |   .8788689    .0221461    39.68   0.000     .8354633    .9222745
       _cons |   3.446075    .8868795     3.89   0.000     1.707823    5.184327
-------------+----------------------------------------------------------------
    /sigma_u |   .4981939    .3098616                      .1472234    1.685855
    /sigma_e |   4.844088    .1512218                      4.556584    5.149732
         rho |   .0104665    .0129524                      .0006308    .0816856
------------------------------------------------------------------------------
Likelihood-ratio test of sigma_u=0: chibar2(01)=     1.19 Prob>=chibar2 = 0.138

. test 1+laggdp = -lag2gdp

 ( 1)  [Egdp]lag2gdp + [Egdp]laggdp = -1

           chi2(  1) =    7.47
         Prob > chi2 =    0.0063

. estat ic

Akaike's information criterion and Bayesian information criterion

-----------------------------------------------------------------------------
       Model |    Obs    ll(null)   ll(model)     df          AIC         BIC
-------------+---------------------------------------------------------------
           . |    522   -2186.504   -1566.429      5     3142.858    3164.146
-----------------------------------------------------------------------------
```

Experiment 1 — Output Learning Rule (5) — Experiment 2

```
Random-effects ML regression                    Number of obs      =       546
Group variable: id                              Number of groups   =        13

Random effects u_i ~ Gaussian                   Obs per group: min =        42
                                                               avg =      42.0
                                                               max =        42

                                                LR chi2(2)         =   1191.38
Log likelihood  = -1565.1197                    Prob > chi2        =    0.0000

------------------------------------------------------------------------------
        Egdp |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      laginf |  -.4816517    .2156126    -2.23   0.025    -.9042446   -.0590587
      laggdp |   .9695792    .0157113    61.71   0.000     .9387857    1.000373
       _cons |   1.164832    .6124559     1.90   0.057    -.0355592    2.365224
-------------+----------------------------------------------------------------
    /sigma_u |   .9795257    .2762163                      .5636191    1.702339
    /sigma_e |   4.192914    .1284142                      3.948632    4.452308
         rho |   .0517514    .0279803                      .0159582    .1333377
------------------------------------------------------------------------------
Likelihood-ratio test of sigma_u=0: chibar2(01)=    13.52 Prob>=chibar2 = 0.000

.  estat ic

Akaike's information criterion and Bayesian information criterion

-----------------------------------------------------------------------------
       Model |    Obs    ll(null)   ll(model)     df          AIC         BIC
-------------+---------------------------------------------------------------
           . |    546   -2160.809    -1565.12      5     3140.239    3161.752
-----------------------------------------------------------------------------
```

```
Random-effects ML regression                    Number of obs      =       522
Group variable: id                              Number of groups   =         9

Random effects u_i ~ Gaussian                   Obs per group: min =        58
                                                               avg =      58.0
                                                               max =        58

                                                LR chi2(2)         =   1241.62
Log likelihood  = -1565.6969                    Prob > chi2        =    0.0000

------------------------------------------------------------------------------
        Egdp |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      laginf |  -.7941109    .2690447    -2.95   0.003    -1.321429   -.2667931
      laggdp |   .8912103    .0177632    50.17   0.000     .8563951    .9260255
       _cons |   3.146633    .7203819     4.37   0.000      1.73471    4.558555
-------------+----------------------------------------------------------------
    /sigma_u |   .4993496    .3091387                      .1484023     1.68023
    /sigma_e |   4.837181    .1510061                      4.550087     5.14239
         rho |   .0105444    .0129874                      .0006447    .0815894
------------------------------------------------------------------------------
Likelihood-ratio test of sigma_u=0: chibar2(01)=     1.20 Prob>=chibar2 = 0.136

.  estat ic

Akaike's information criterion and Bayesian information criterion

-----------------------------------------------------------------------------
       Model |    Obs    ll(null)   ll(model)     df          AIC         BIC
-------------+---------------------------------------------------------------
           . |    522   -2186.504   -1565.697      5     3141.394    3162.682
-----------------------------------------------------------------------------
```

Experiment 1 — Output Learning Rule (6) — Experiment 2

Experiment 1

```
Random-effects ML regression                    Number of obs     =        546
Group variable: id                              Number of groups  =         13

Random effects u_i ~ Gaussian                   Obs per group: min =        42
                                                               avg =      42.0
                                                               max =        42

                                                LR chi2(1)        =    1186.41
Log likelihood  = -1567.6031                    Prob > chi2       =     0.0000

------------------------------------------------------------------------------
        Egdp |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      laggdp |   .9813751   .0148666    66.01   0.000      .952237    1.010513
       _cons |   .0522006   .3566836     0.15   0.884    -.6468864    .7512876
-------------+----------------------------------------------------------------
    /sigma_u |   .9775233   .2767889                      .5611858    1.702737
    /sigma_e |   4.212495   .1290179                      3.967065     4.47311
         rho |   .0510972   .0277591                      .0156676    .1322489
------------------------------------------------------------------------------
Likelihood-ratio test of sigma_u=0: chibar2(01)=    13.27 Prob>=chibar2 = 0.000

. estat ic

Akaike's information criterion and Bayesian information criterion

-----------------------------------------------------------------------------
       Model |        Obs  ll(null)  ll(model)      df         AIC        BIC
-------------+---------------------------------------------------------------
           . |        546 -2160.809  -1567.603       4    3143.206   3160.417
-----------------------------------------------------------------------------
```

Experiment 2

```
Random-effects ML regression                    Number of obs     =        522
Group variable: id                              Number of groups  =          9

Random effects u_i ~ Gaussian                   Obs per group: min =        58
                                                               avg =      58.0
                                                               max =        58

                                                LR chi2(1)        =    1232.98
Log likelihood  = -1570.0161                    Prob > chi2       =     0.0000

------------------------------------------------------------------------------
        Egdp |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      laggdp |   .9271646    .013038    71.11   0.000     .9016106    .9527186
       _cons |   1.175289   .2699576     4.35   0.000     .6461819    1.704396
-------------+----------------------------------------------------------------
    /sigma_u |   .4924423   .3135166                      .1413944    1.715056
    /sigma_e |    4.87808   .1522876                       4.58855    5.185879
         rho |   .0100881   .0127826                      .0005652    .0822325
------------------------------------------------------------------------------
Likelihood-ratio test of sigma_u=0: chibar2(01)=     1.11 Prob>=chibar2 = 0.146

. estat ic

Akaike's information criterion and Bayesian information criterion

-----------------------------------------------------------------------------
       Model |        Obs  ll(null)  ll(model)      df         AIC        BIC
-------------+---------------------------------------------------------------
           . |        522 -2186.504  -1570.016       4    3148.032   3165.063
-----------------------------------------------------------------------------
```

Experiment 1 — Output Learning Rule (7) — Experiment 2

Experiment 1

```
Random-effects ML regression                    Number of obs     =        546
Group variable: id                              Number of groups  =         13

Random effects u_i ~ Gaussian                   Obs per group: min =        42
                                                               avg =      42.0
                                                               max =        42

                                                LR chi2(1)        =      69.49
Log likelihood  = -2126.0628                    Prob > chi2       =     0.0000

------------------------------------------------------------------------------
        Egdp |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      laginf |   -4.95368    .575431    -8.61   0.000    -6.081504   -3.825856
       _cons |   19.92242   1.295164    15.38   0.000     17.38394    22.46089
-------------+----------------------------------------------------------------
    /sigma_u |          0   .6619722                             .           .
    /sigma_e |   11.88111    .359531                      11.19693    12.60709
         rho |          0  (omitted)
------------------------------------------------------------------------------
Likelihood-ratio test of sigma_u=0: chibar2(01)=     0.00 Prob>=chibar2 = 1.000

.  estat ic

Akaike's information criterion and Bayesian information criterion

-----------------------------------------------------------------------------
       Model |        Obs  ll(null)  ll(model)      df         AIC        BIC
-------------+---------------------------------------------------------------
           . |        546 -2160.809  -2126.063       4    4260.126   4277.336
-----------------------------------------------------------------------------
```

Experiment 2

```
Random-effects ML regression                    Number of obs     =        522
Group variable: id                              Number of groups  =          9

Random effects u_i ~ Gaussian                   Obs per group: min =        58
                                                               avg =      58.0
                                                               max =        58

                                                LR chi2(1)        =     325.41
Log likelihood  = -2023.7986                    Prob > chi2       =     0.0000

------------------------------------------------------------------------------
        Egdp |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      laginf |  -10.05082   .4729247   -21.25   0.000    -10.97774   -9.123906
       _cons |   26.80021    1.25255    21.40   0.000     24.34526    29.25517
-------------+----------------------------------------------------------------
    /sigma_u |          0   .6015132                             .           .
    /sigma_e |   11.68225   .3615481                      10.99469     12.4128
         rho |          0  (omitted)
------------------------------------------------------------------------------
Likelihood-ratio test of sigma_u=0: chibar2(01)=     0.00 Prob>=chibar2 = 1.000

.  estat ic

Akaike's information criterion and Bayesian information criterion

-----------------------------------------------------------------------------
       Model |        Obs  ll(null)  ll(model)      df         AIC        BIC
-------------+---------------------------------------------------------------
           . |        522 -2186.504  -2023.799       4    4055.597   4072.628
-----------------------------------------------------------------------------
```